# Demand Response of Data Centers: A Real-time Pricing Game between Utilities in Smart Grid

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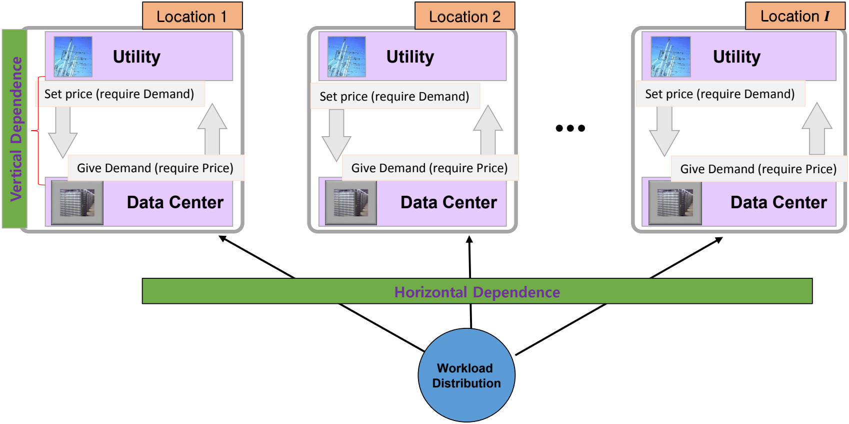
# Motivation

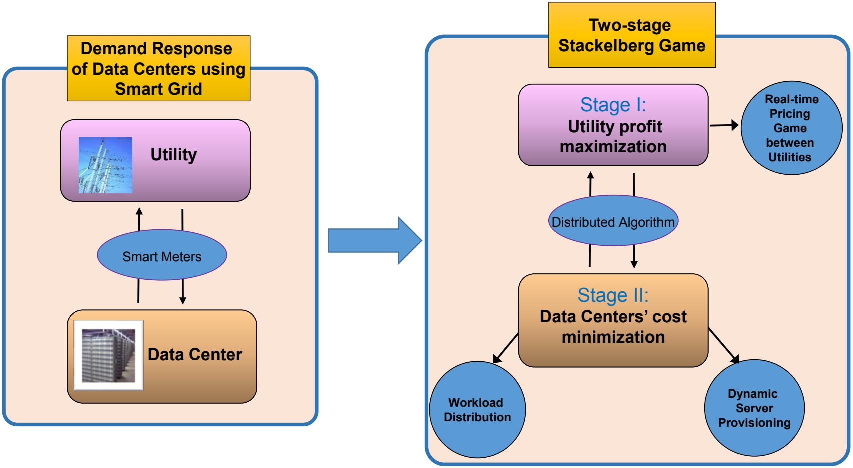
1. DCs consumed 1.5% of the worldwide electricity supply in 2011 and this fraction is expected to grow to 8% by 2020
2. DC operators paid more than $10M (Qureshi 2009)
3. DC operators can save 5% – 45% cost by leveraging time and location diversities of prices

- The electricity price applying on DC does not change with demand
- Demand Response of Data Centers: receiving consideration
- Pricing for DR: a *right price* not only at the *right time* but also on the *right amount* of demand

# Challenges


Location 1
Utility
Set price (require Demand)
Give Demand (require Price)
Data Center
Vertical Dependence
Location 2
Utility
Set price (require Demand)
Give Demand (require Price)
Data Center
Location $I$
Utility
Set price (require Demand)
Give Demand (require Price)
Data Center
Horizontal Dependence
Workload Distribution

# Approaches


Demand Response of Data Centers using Smart Grid
Utility
Smart Meters
Data Center
Two-stage Stackelberg Game
Stage I:
Utility profit maximization
Real-time Pricing Game between Utilities
Distributed Algorithm
Stage II:
Data Centers' cost minimization
Workload Distribution
Dynamic Server Provisioning

# Stage II: DCs' Cost Minimization

## Optimization Problem

$$\text{DC}: \quad \text{minimize} \quad \sum_{t=1}^{T}\sum_{i=1}^{I} e_i(t)p_i(t) + \omega d_i \lambda_i^2(t) \tag{1}$$

$$\text{subject to} \quad \frac{1}{s_i(t)\mu_i - \lambda_i(t)} + d_i \leq D_i, \forall i \tag{2}$$

$$\sum_{i=1}^{I} \lambda_i(t) = \Lambda(t), \ \forall t, \tag{3}$$

$$0 \leq s_i(t) \leq S_i, \ \forall i, t, \tag{4}$$

$$0 \leq \lambda_i(t) \leq s_i(t)\mu_i, \ \forall i, t, \tag{5}$$

$$\text{variables} \quad s_i(t), \lambda_i(t), \ \forall i, t. \tag{6}$$

# Stage I: Utility Revenue and Cost

**Revenue**

$$\mathcal{R}_i(p(t)) = \big(e_i(p(t)) + B_i(p_i(t))\big)\, p_i(t)$$

**Cost**

$$\mathcal{C}_i(p(t)) = \gamma ELI = \gamma \left(\frac{e_i(p(t)) + B_i(p_i(t))}{C_i(t)}\right)^2 C_i(t),$$

# Stage-I: A Non-Cooperative Pricing Game Formulation

- *Players*: the utilities in the set $\mathcal{I}$;
- *Strategy*: $p_i^l \leq p_i(t) \leq p_i^u$, $\forall i \in \mathcal{I}, t \in \mathcal{T}$;
- *Payoff function*: $\sum_{t=1}^{T} u_i(p_i(t), p_{-i}(t))$, $\forall i \in \mathcal{I}$.

$$u_i(p_i(t), p_{-i}(t)) = \mathcal{R}_i(p(t)) - \mathcal{C}_i(p(t)),$$

# Backward Induction: Optimal Solutions at Stage II

Observe that the QoS constraint must be active

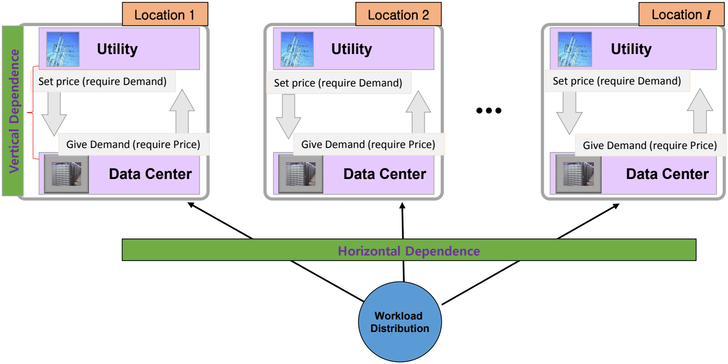
$$s_i(\lambda_i) = \left[\frac{1}{\mu_i}\left(\lambda_i + \tilde{D}_i^{-1}\right)\right]_0^{S_i},$$

## Optimization Problem

$$\begin{aligned}
\text{DC}': \quad \min_{\lambda} \quad & \sum_{i=1}^{I} f_i(\lambda_i) && (7)\\
\text{s.t.} \quad & \sum_{i=1}^{I} \lambda_i = \Lambda, && (8)\\
& \lambda_i \geq 0, \ \forall i, && (9)
\end{aligned}$$

$$f_i(\lambda_i) := \omega d_i \lambda_i^2 + p_i\left(a_i + \frac{b_i}{\mu_i}\right)\lambda_i + p_i\left(e_b + \frac{b_i \tilde{D}_i^{-1}}{\mu_i}\right)$$

# Backward Induction: Optimal Solutions at Stage II




$$e_i^*(p) = \frac{A_i^2 p_i}{2\omega d_i}\left(\frac{1}{\hat{d} d_i} - 1\right) + \frac{A_i}{2\omega \hat{d} d_i}\sum_{j\neq i}\frac{A_j p_j}{d_j} + \frac{A_i \Lambda}{\hat{d} d_i} + \frac{b_i}{\mu_i \tilde{D}_i} + e_i^b.$$

# Backward Induction: Nash equilibrium at Stage I

Existence: concave game (Rosen 1965)

Best response updates

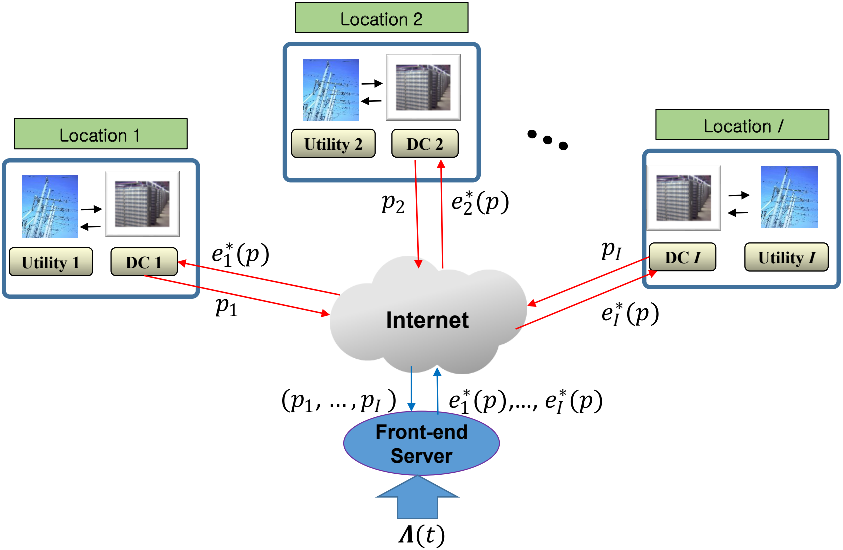
$$p_i^{(k+1)} = \mathcal{BR}_i\left(p_{-i}^{(k)}\right) = \left[\frac{1/2 - \gamma N_i/C_i}{1 - \gamma N_i/C_i} \frac{h\left(p_{-i}^{(k)}\right)}{(-N_i)}\right]_{\mathcal{P}_i}, \forall i$$

Uniqueness: $p_i^e = \mathcal{BR}_i(p_{-i}^e), \forall i$

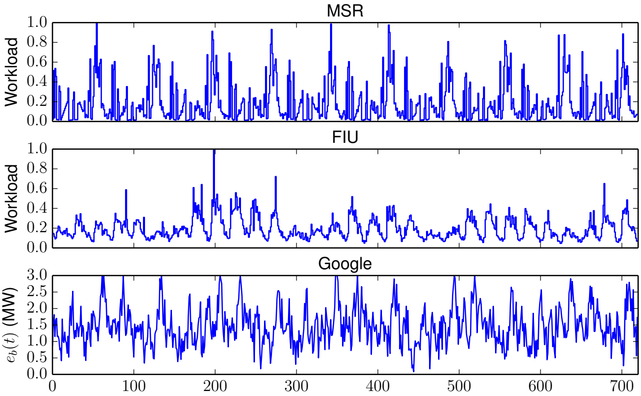
Condition

$$\omega \geq \max_i \left\{ \frac{A_i \sum_{j \neq i} A_j/d_j - A_i^2 \hat{d}(1 - 1/(d_i \hat{d}))}{2\beta_i \hat{d} d_i} \right\},$$

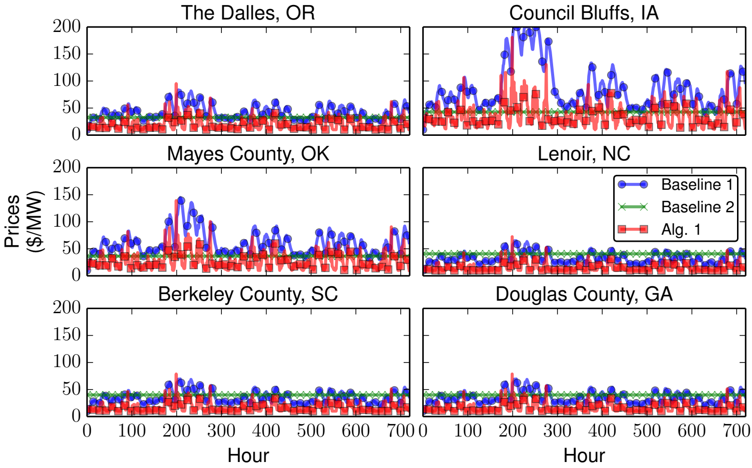
# Distributed Algorithm



# Trace-based Simulations

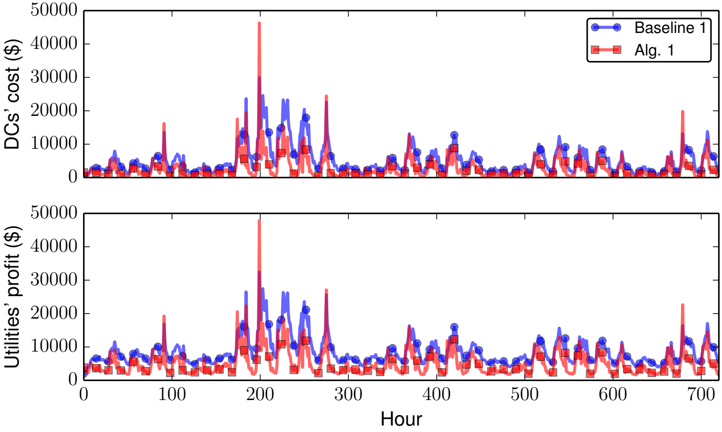


# Dynamic Prices



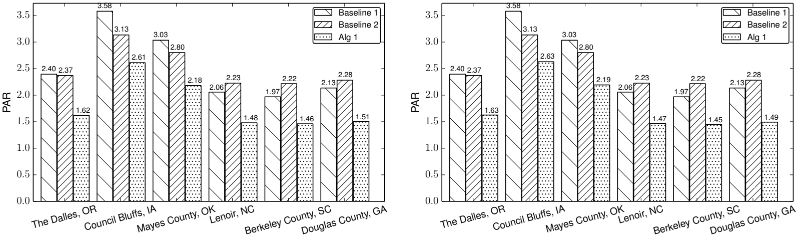

(a) FIU trace

# DC's cost and utilities' profit




(a) FIU trace

# PAR




(a) $\gamma = 1$, (b) $\gamma = 4$

# Summary and Future Work

## Summary

- DR of DCs: interactions between DCs and utilities via pricing
- Two-stage Stackelberg game: utilities are leaders, DCs are follows
- Flatten the demand over time and space

## Future Work

- Deadline constraint
- Workload estimation errors
- Risk consideration

# Q&A

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