Locally Differentially Private Protocols for Frequency Estimation

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Differential Privacy

Differential Privacy

Classical setting

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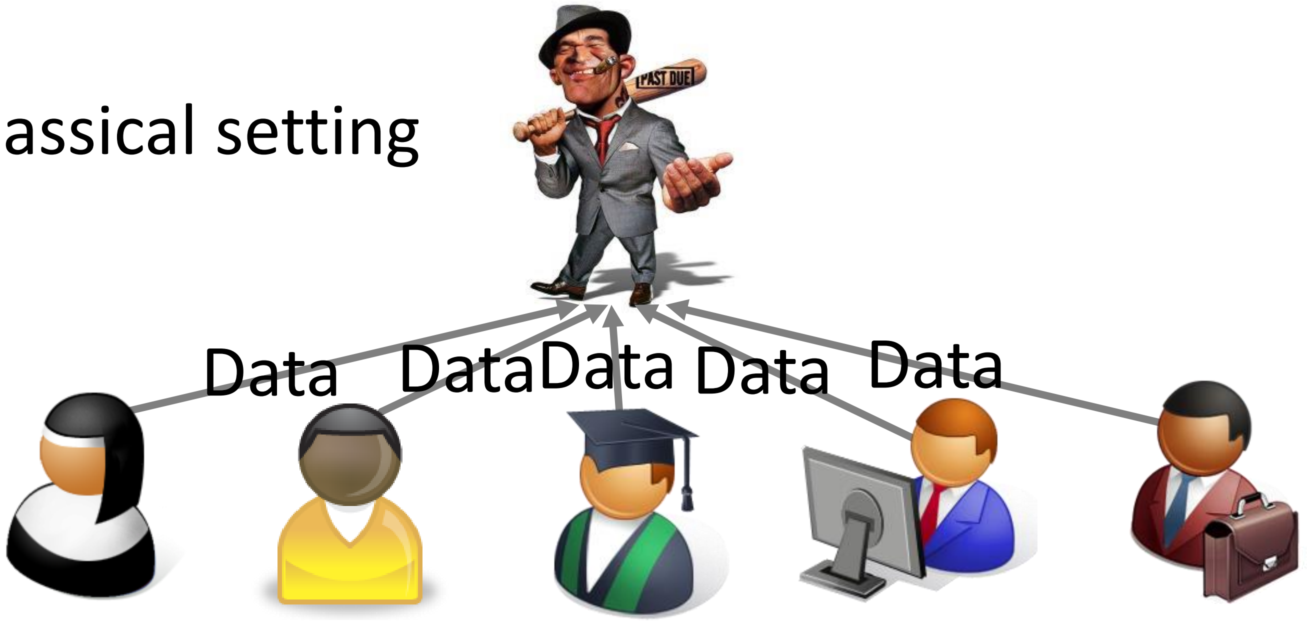
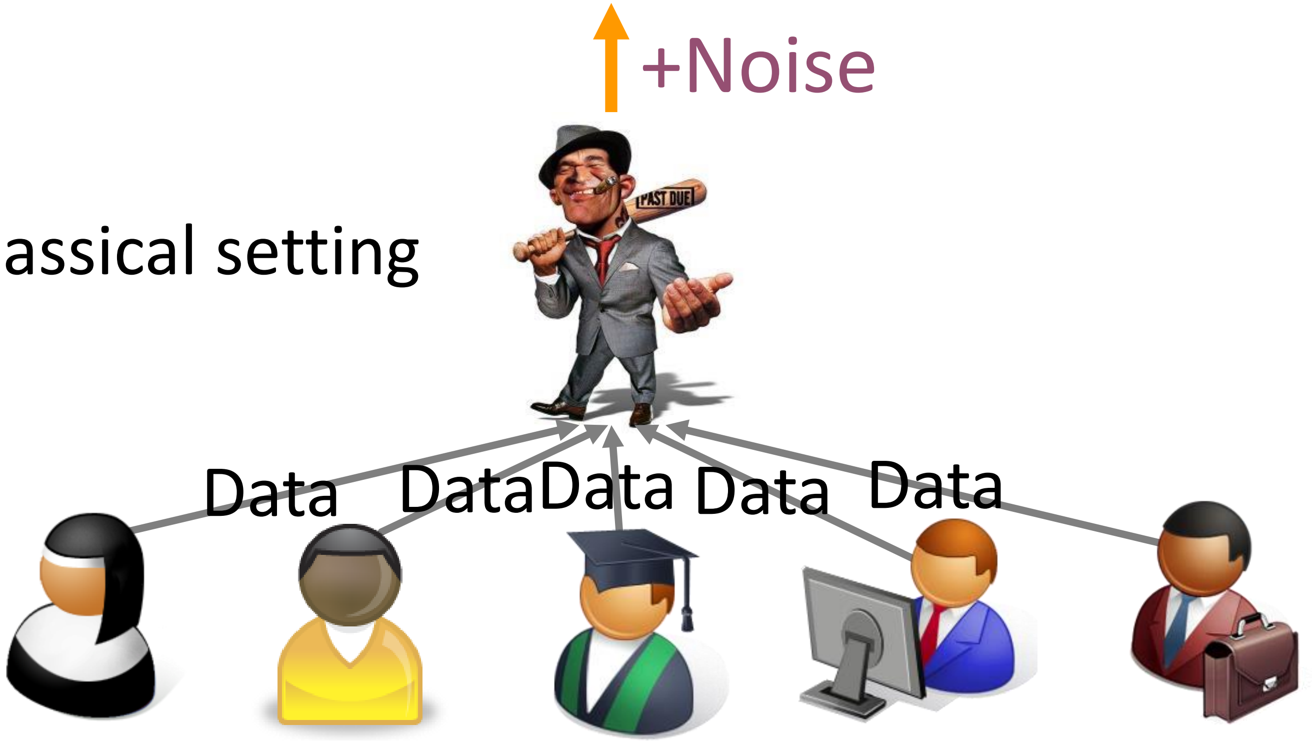


Differential Privacy

Classical setting



Differential Privacy

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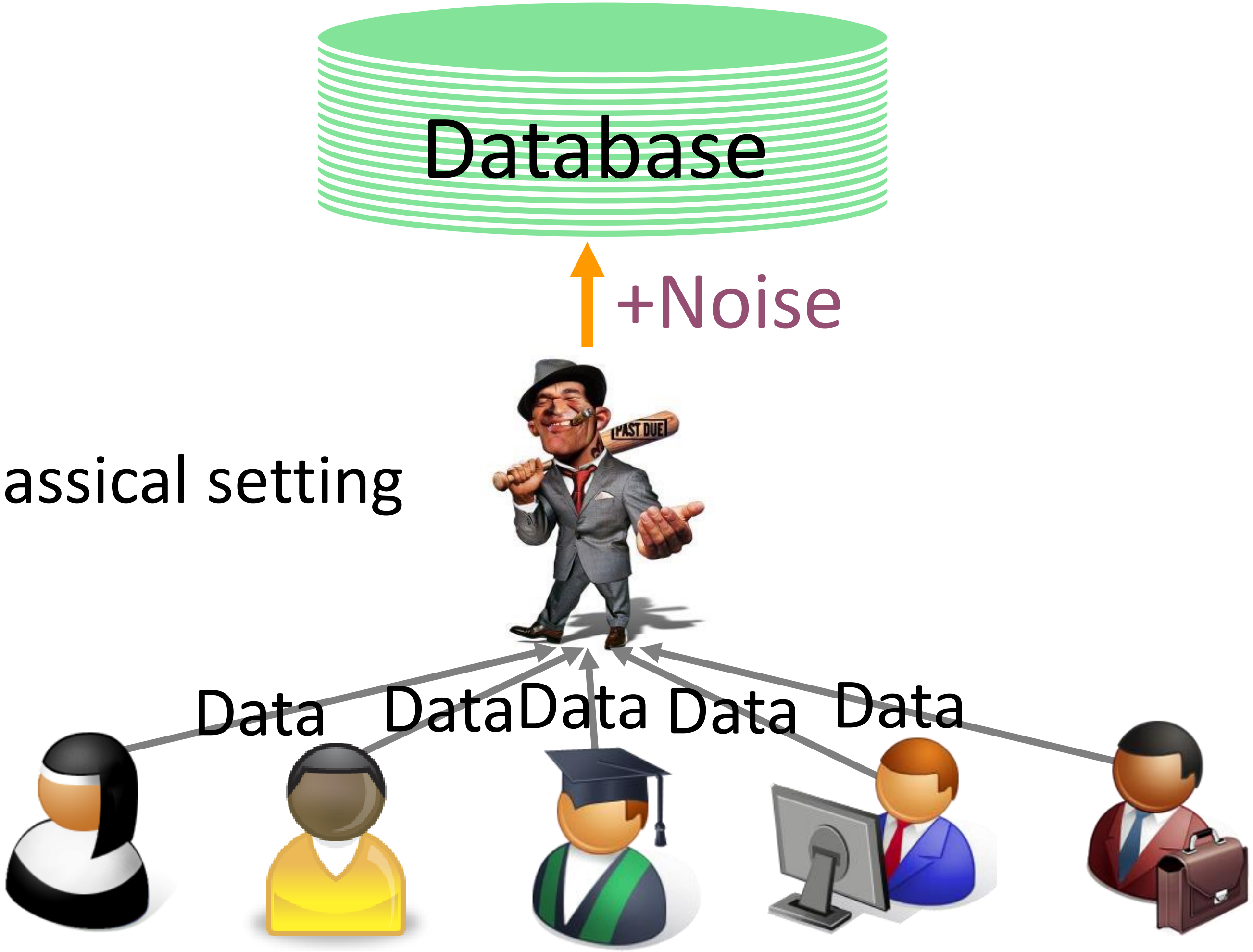

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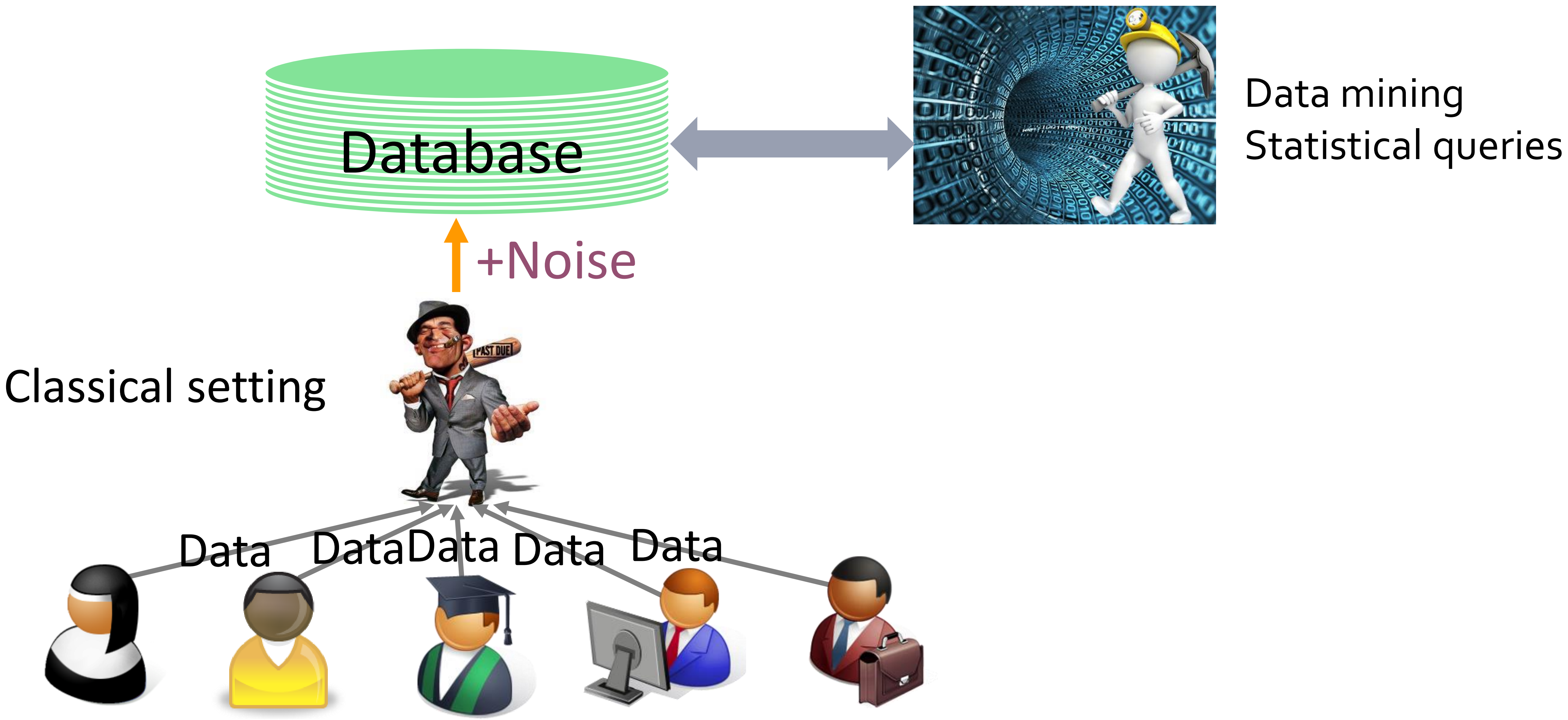


Differential Privacy

Database

+Noise

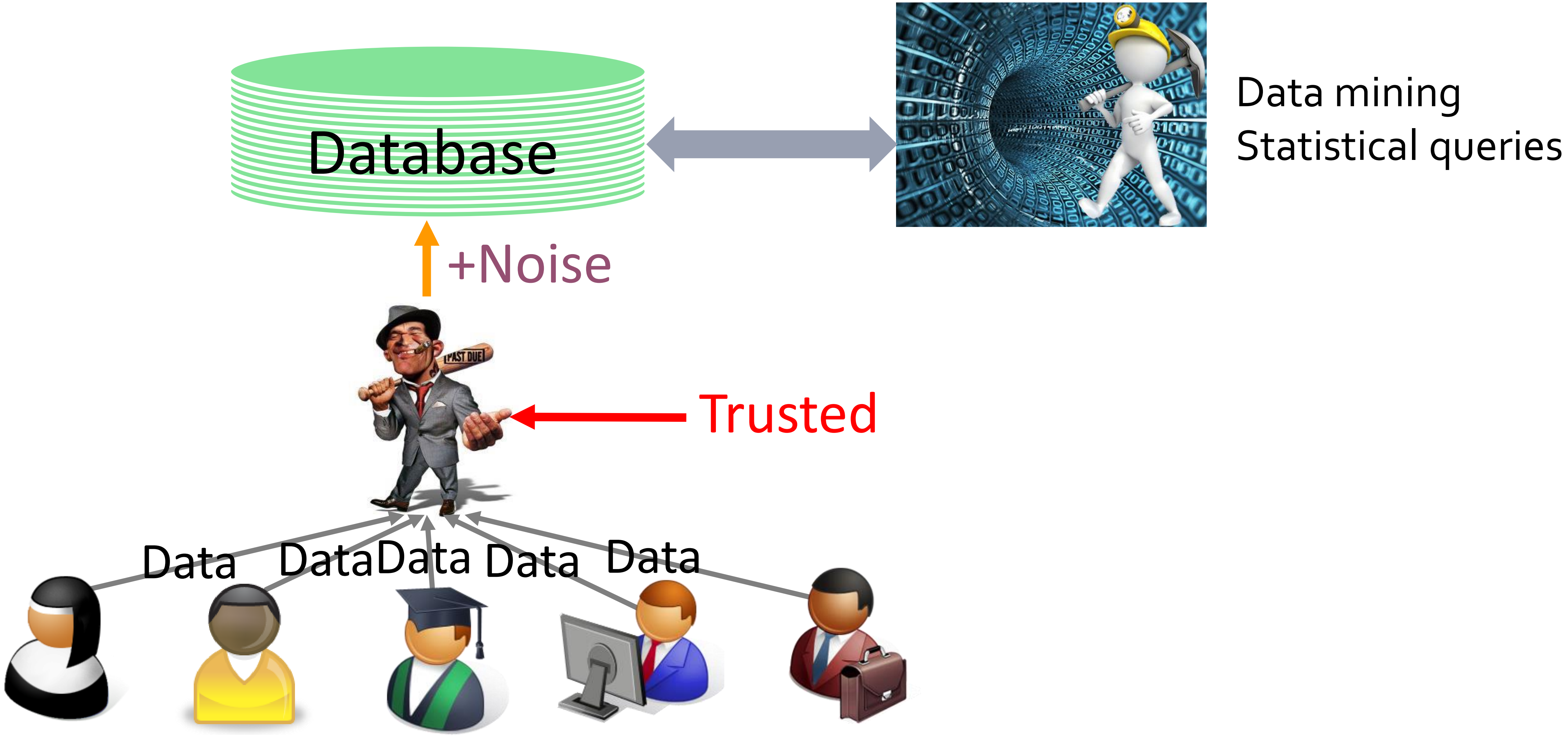
Data mining
Statistical queries

Classical setting

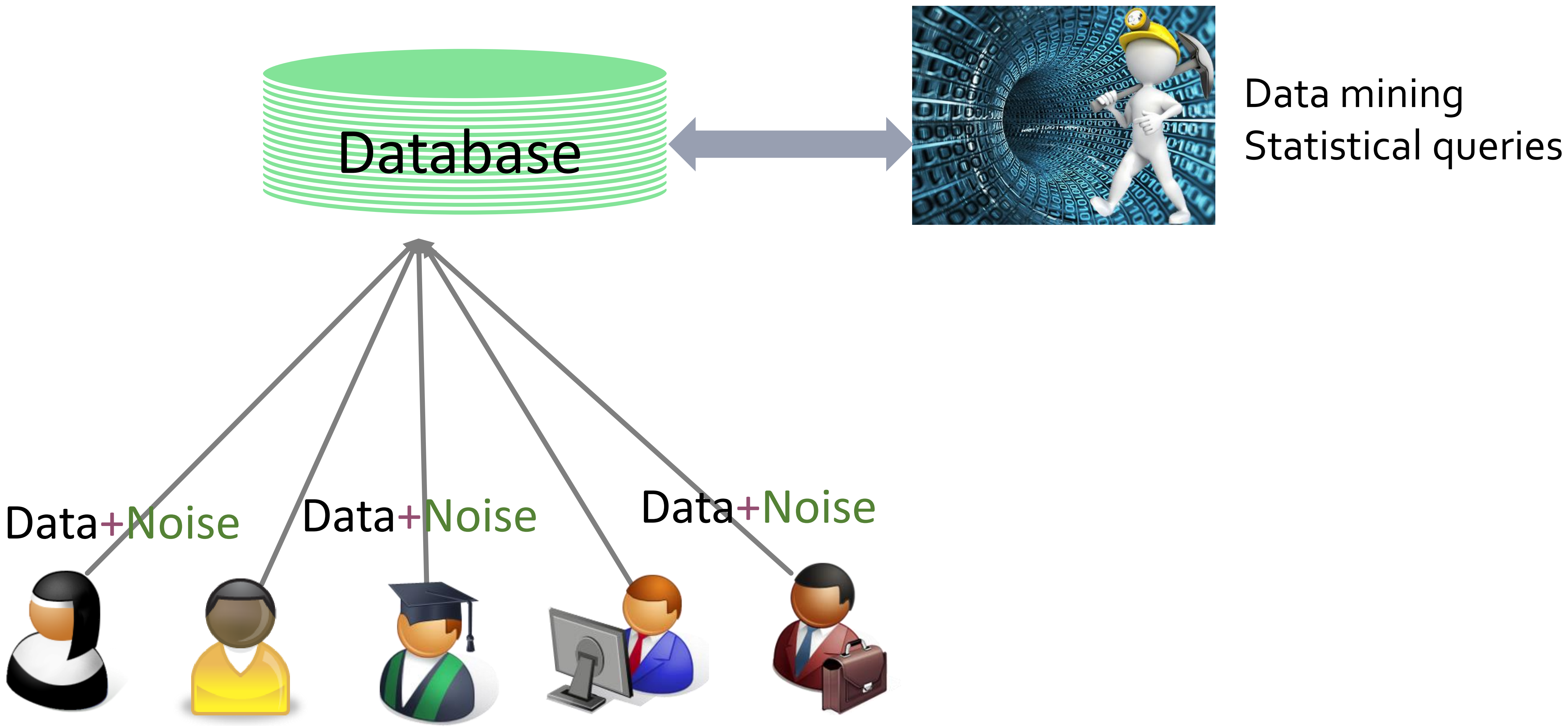
Differential Privacy Interpretation:
The decision to include/exclude individual’s record has minimal (ε) influence on the outcome.
Smaller ε ➔ Stronger Privacy

Data

Differential Privacy



Local Differential Privacy



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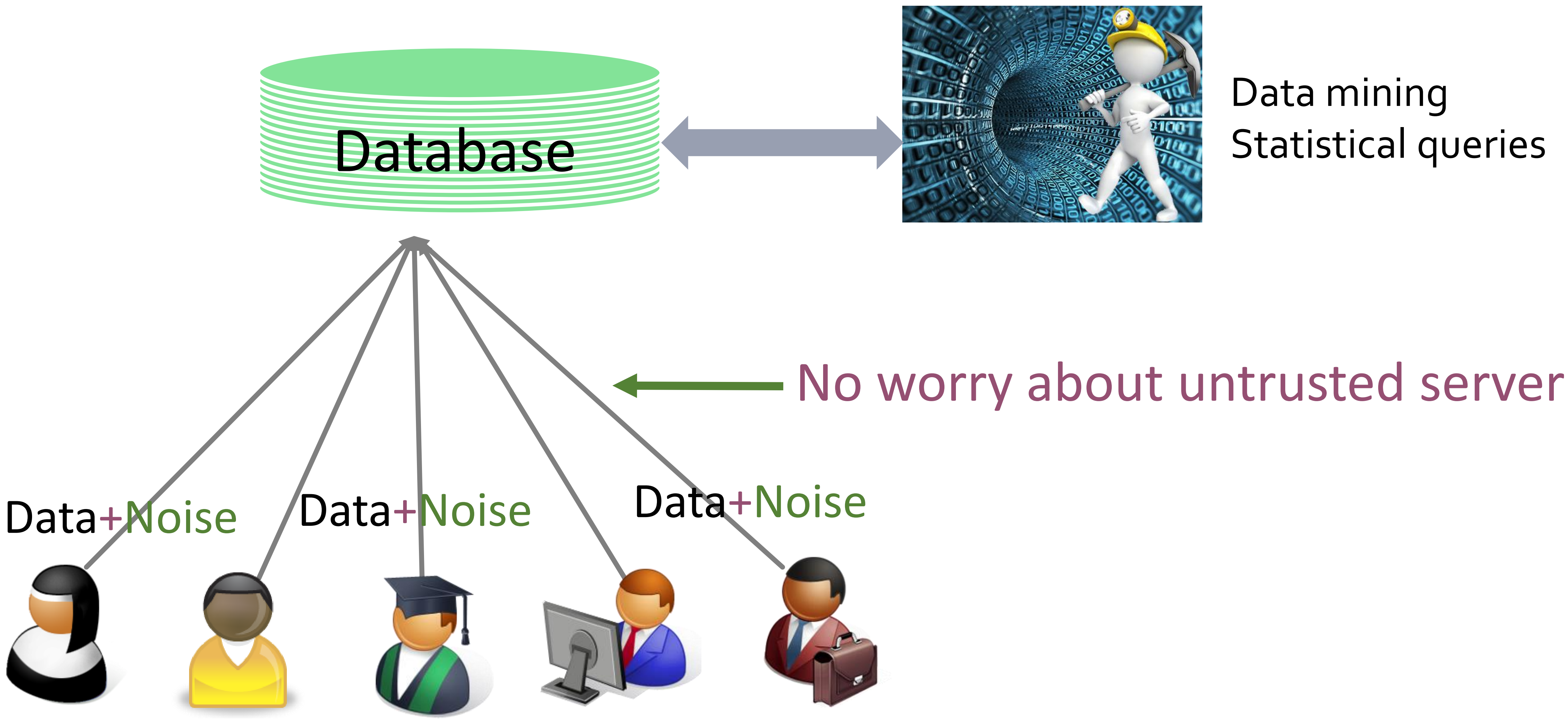

Local Differential Privacy

RAPPOR: Randomized Aggregatable Privacy-Preserving Ordinal Response

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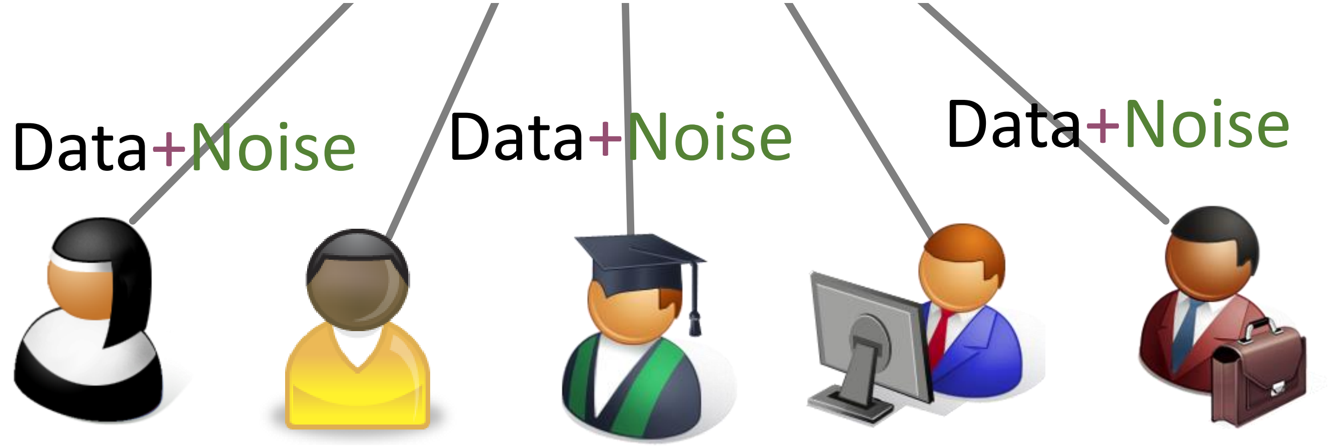
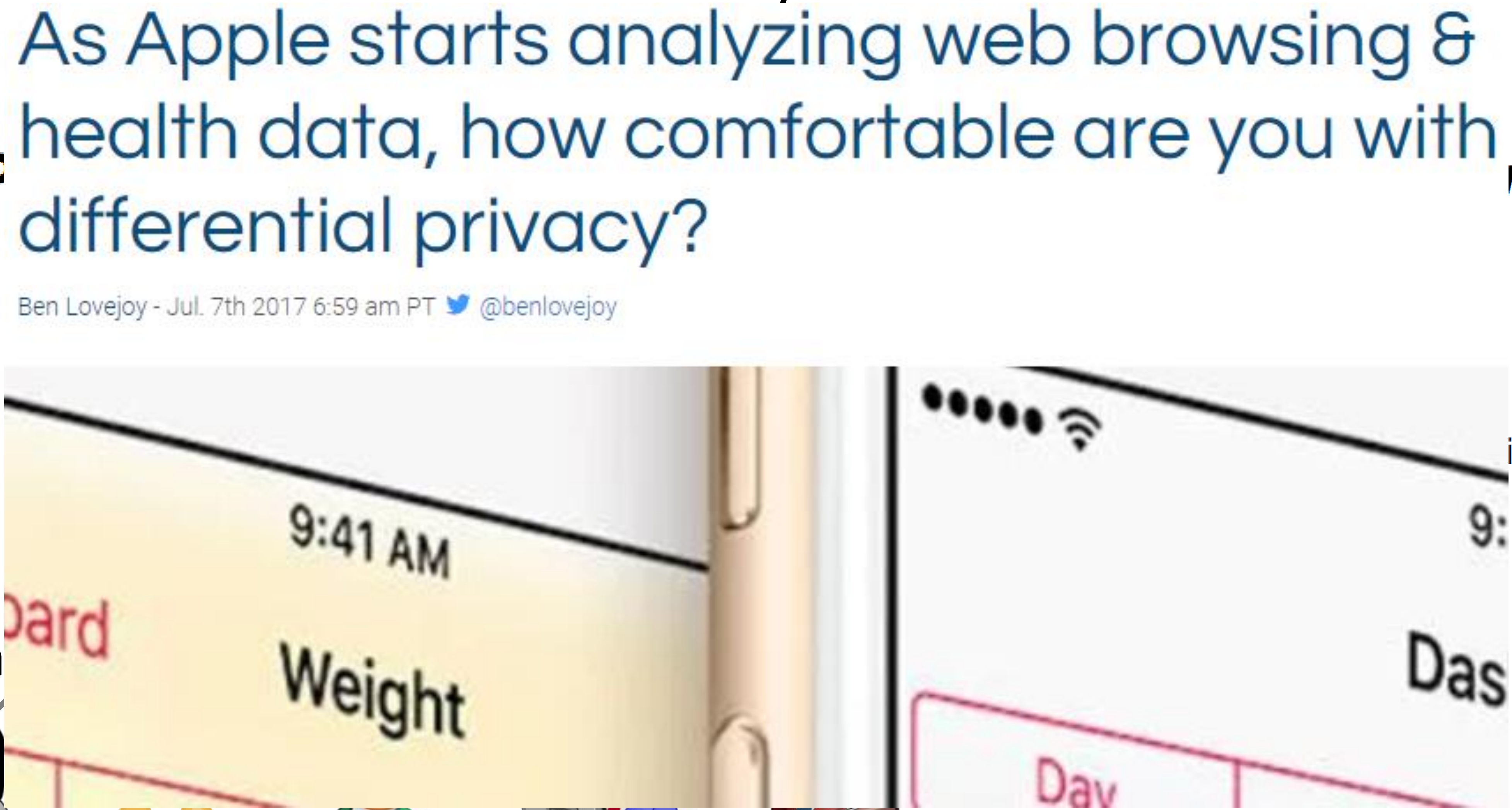


Local Differential Privacy

RAP ving



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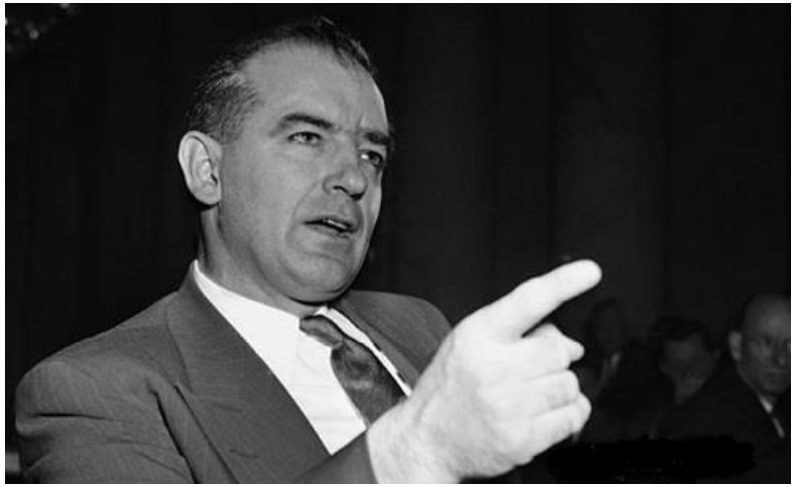
Data

The Warner Model (1965)



- Survey technique for private questions
- Survey people:
  - "Are you communist party?"
- Each person:
  - Flip a secret coin
  - Answer truth if head (w/p 0.5)
  - Answer randomly if tail
  - E.g., a communist will answer "yes" w/p 75%, and "no" w/p 25%

Provide deniability:
Seeing answer, not certain about the secret.

- To get unbiased estimation of the distribution:
  - If n_v out of n people are communist, we expect to see $E[\,I_v] = 0.75n_v + 0.25(n - n_v)$ "yes" answers
  - $c(n_v) = \frac{I_v - 0.25n}{0.5}$ is the unbiased estimation of number of communists
  - Since $E[c(n_v)] = \frac{E[I_v] - 0.25n}{0.5} = n_v$

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...tain about the secret.

We say the protocol is ε -LDP iff for any $\boldsymbol{v}$ and $\boldsymbol{v}'$ from "yes" and "no",

$$\frac{\Pr[P(\boldsymbol{v}) = \boldsymbol{v}]}{\Pr[P(\boldsymbol{v}') = \boldsymbol{v}]} \le e^{\varepsilon}$$

This only handles binary attribute. We want to handle the more general setting.

Abstract LDP Protocol

- $x \coloneqq E(v)$
takes input value v from domain D and outputs an encoded value x
- $y \coloneqq P(x)$
takes an encoded value x and outputs y.

P satisfies ε-LDP

$\xrightarrow{y}$

- $c \coloneqq Est(\{y\})$
takes reports $\{y\}$ from all users and outputs estimations $c(v)$ for any value v in domain D

We focus on frequency estimation

Frequency Estimation Protocols

- Direct Encoding (Generalized Random Response) [Warner'65]
  - Generalize binary attribute to arbitrary domain
- Unary Encoding (Basic One-time RAPPOR) [Erlingsson et al'14]
  - Encode into a bit-vector and perturb each bit
- Binary Local Hash [Bassily and Smith'15]
  - Encode by hashing and then perturb

Direct Encoding (Random Response)

- User:
  - Encode $x = v$ (suppose v from $D = \{1,2, \dots, d\}$)
  - Toss a coin with bias p
  - If it is head, report the true value $y = x$
  - Otherwise, report any other value with probability $q = \frac{1-p}{d-1}$ (uniformly at random)
    - $p = \frac{e^{\varepsilon}}{e^{\varepsilon}+d-1}, q = \frac{1}{e^{\varepsilon}+d-1} \Rightarrow \frac{\Pr[P(v)=v]}{\Pr[P(v\prime)=v]} = \frac{p}{q} = e^{\varepsilon}$
- Aggregator:
  - Suppose n_v users possess value v, I_v is the number of reports on v.
  - $E[I_v] = n_v \cdot p + (n - n_v) \cdot q$
  - Unbiased Estimation: $c(v) = \frac{I_v - n \cdot q}{p-q}$

Direct Encoding (Random Response)

- User:
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  - Toss a coin with bias p
  - If it is head, report the true value $y = x$
  - Otherwise, … ly at random)

Intuitively, the higher p, the more accurate

    - $p = \frac{e^{\varepsilon}}{e^{\varepsilon}+d-1}, q = \frac{1}{e^{\varepsilon}+d-1} \Rightarrow \frac{\Pr[P(v)=v]}{\Pr[P(v\prime)=v]} = \frac{p}{q} = e^{\varepsilon}$

- Aggregator:

However, when d is large, p becomes small

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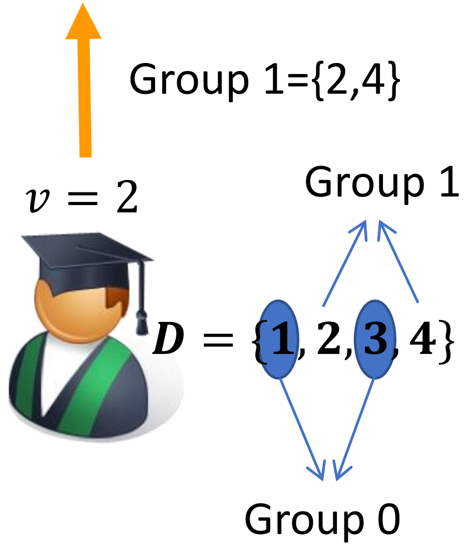
Unary Encoding (Basic RAPPOR)

- Encode the value v into a bit string $\boldsymbol{x} \coloneqq \vec{0}, \boldsymbol{x}[v] \coloneqq 1$
  - e.g., $D = \{1,2,3,4\}, v = 3,$ then $\boldsymbol{x} =$ [0,0,1,0]
- Perturb each bit independently
  - $p = \frac{e^{\varepsilon/2}}{e^{\varepsilon/2}+1}, q = \frac{1}{e^{\varepsilon/2}+1} \Rightarrow \frac{\Pr[P(E(v))=\boldsymbol{x}]}{\Pr[P(E(v'))=\boldsymbol{x}]} = \frac{\prod_i \Pr[\boldsymbol{x}[i]|v]}{\prod_i \Pr[\boldsymbol{x}[i]|v\prime]} = \frac{p\cdot(1-q)}{q\cdot(1-p)} = e^{\varepsilon}$
    - Since $\boldsymbol{x}$ is unary encoding of v, $\boldsymbol{x}$ and $\boldsymbol{x}'$ differ in two locations
- Intuition:
  - By unary encoding, each location can only be 0 or 1, effectively reducing d in each location to 2.
  - When d is large, UE is better.
- To estimate frequency of each value, do it for each bit.

Binary Local Hash

- The protocol description itself is more complicated
- Now we describe a simpler equivalent
- Each user uses a random hash function from D to $\{0,1\}$
- The user then perturbs the bit with probabilities
  - $p = \frac{e^{\varepsilon}}{e^{\varepsilon}+g-1} = \frac{e^{\varepsilon}}{e^{\varepsilon}+1}, q = \frac{1}{e^{\varepsilon}+g-1} = \frac{1}{e^{\varepsilon}+1} \Rightarrow \frac{\Pr[P(E(\boldsymbol{v}))=b]}{\Pr[P(E(\boldsymbol{v'}))=b]} = \frac{p}{q} = e^{\varepsilon}$
- The user then reports the bit and the hash function
- The aggregator increments the reported group
- $E[I_v] = n_v \cdot p + (n - n_v) \cdot (\frac{1}{2}q + \frac{1}{2}p)$
- Unbiased Estimation: $c(v) = \frac{I_v - n \cdot \frac{1}{2}}{p - \frac{1}{2}}$

$+1 \quad +1$
$[\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}]$



Takeaway

- Key Question:
  - Maximize utility of frequency estimation under LDP
- Key Idea:
  - A framework to generalize and optimize these protocols
- Results:
  - Optimized Unary Encoding and Local Hash
  - By improving the frequency estimator, results in other more complicated settings that use LDP can be improved, e.g., private learning, frequent itemset mining, etc.

Method

- We measure utility of a mechanism by its variance
  - E.g., in Random Response, $Var[c(v)] = Var\left[\frac{I_v - n\cdot q}{p-q}\right] = \frac{Var[I_v]}{(p-q)^2} \approx \frac{n\cdot q\cdot(1-q)}{(p-q)^2}$
- We propose a framework called 'pure' and cast existing mechanisms into the framework.
  - For each output y, define a set of input v called Support
    - Intuition: each output votes for a set of input
    - After perturbation, output y will support input from Support(y)
    - E.g., In BLH, Support(y) = $\{v|H(v) = y\}$
  - Define p' and q' such that $P(E(v))$ support v w/p p' and don't w/p q'
    - E.g., In Random Response, $p' = p$, $q' = q$
- Pure means this holds for all input-output pairs

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- Pure means this holds for all input-output pairs

$$min_{q'} Var[c(v)]$$

or $min_{q'} \frac{n\cdot q'\cdot(1-q')}{(p'-q)'^2}$

where p', q' satisfy ε-LDP

Optimized UE

- In the original UE, each bit is perturbed independently
  - $p = \frac{e^{\varepsilon/2}}{e^{\varepsilon/2}+1}, q = \frac{1}{e^{\varepsilon/2}+1}$
- We want to make p higher.
- **Key Insight:** We perturb 0 and 1 differently!
- There are more 0, so we perturb with greater p; there is a single 1, so we perturb with smaller p
  - For bit 0: $p_0 = \frac{e^{\varepsilon}}{e^{\varepsilon}+1}, q_0 = \frac{1}{e^{\varepsilon}+1}$
  - For bit 1: $p_1 = \frac{1}{2}, q_1 = \frac{1}{2}$
    - $\Rightarrow \frac{\Pr[P(E(v))=\boldsymbol{x}]}{\Pr[P(E(v'))=\boldsymbol{x}]} = \frac{\prod_i \Pr[\boldsymbol{x}[i]|v]}{\prod_i \Pr[\boldsymbol{x}[i]|v\prime]} = \frac{p_0 \cdot (1-q_0)}{q_0 \cdot (1-p_0)} = e^{\varepsilon}$

Optimized Local Hash (OLH)

- In original BLH, secret is **compressed** into a bit, **perturbed** and transmitted.
  - $p = \frac{e^{\varepsilon}}{e^{\varepsilon}+g-1}, q = \frac{1}{e^{\varepsilon}+g-1} \Rightarrow \frac{\Pr[P(E(v))=\boldsymbol{x}]}{\Pr[P(E(v'))=\boldsymbol{x}]} = \frac{p}{q} = e^{\varepsilon}$ ($g = 2$ groups)
- Two steps that cause information loss:
  - Compressing: loses much
  - Perturbation: pretty accurate
- **Key Insight**: We want to make a balance between the two steps:
  - By compressing into more groups, the first step carries more information
- Variance is optimized when $g = e^{\varepsilon} + 1$
- Read our paper for details!

Comparison of Different Mechanisms

	DE	SHE	THE ($\theta = 1$)	SUE	OUE	BLH	OLH
Communication Cost	$O(\log d)$	$O(d)$	$O(d)$	$O(d)$	$O(d)$	$O(\log n)$	$O(\log n)$
$\mathrm{Var}[\tilde{c}(i)]/n$	$\frac{d-2+e^{\varepsilon}}{(e^{\varepsilon}-1)^2}$	$\frac{8}{\varepsilon^2}$	$\frac{2e^{\varepsilon/2}-1}{(e^{\varepsilon/2}-1)^2}$	$\frac{e^{\varepsilon/2}}{(e^{\varepsilon/2}-1)^2}$	$\frac{4e^{\varepsilon}}{(e^{\varepsilon}-1)^2}$	$\frac{(e^{\varepsilon}+1)^2}{(e^{\varepsilon}-1)^2}$	$\frac{4e^{\varepsilon}}{(e^{\varepsilon}-1)^2}$

Table 1: Com … d Variances for different metho…

Direct Encoding has greater variance with larger d

OUE and OLH have the same variance
But OLH has smaller communication cost

Conclusion

- We survey existing LDP protocols on frequency estimation
- We propose a pure framework and cast existing protocols into it
- We optimize UE and BLH and come up with OUE and OLH

Limitations

- Variance is linear in n, which seems inevitable
  - Therefore, requires large number of users
- Cannot handle large domains

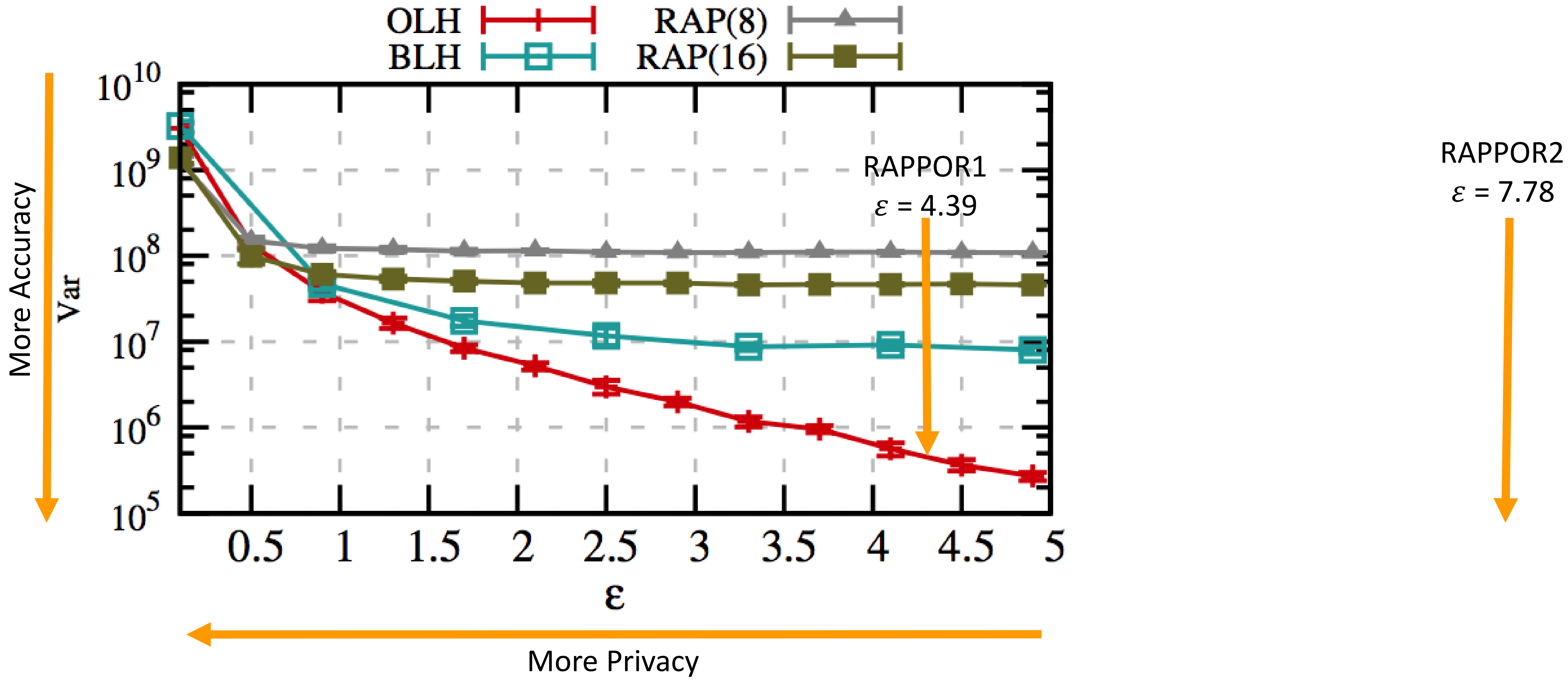
Future Work

- Handling large domains
- Handling set-values

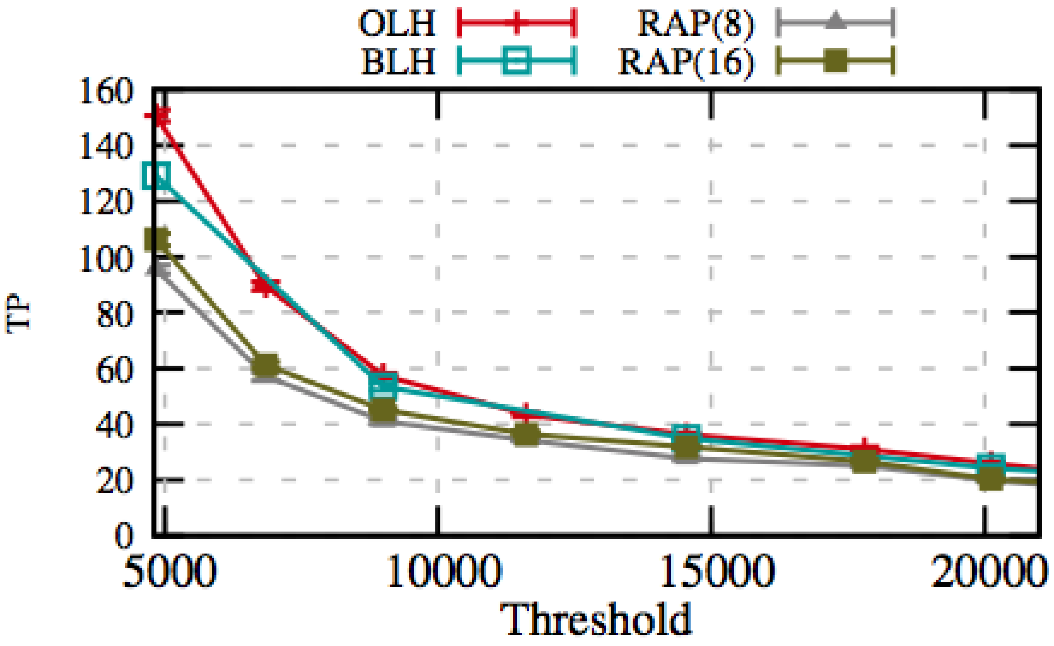
Backup: Experiments Highlights

- Dataset: Kosarak dataset
  - (also on Rockyou dataset and a Synthetic dataset)
- Competitors: RAPPOR, BLH, OLH
  - Randomized Response is not compared because the domain is large
- Key Results:
  - OLH performs magnitudes better, especially when ε is large
  - This also confirms our analytical conclusion

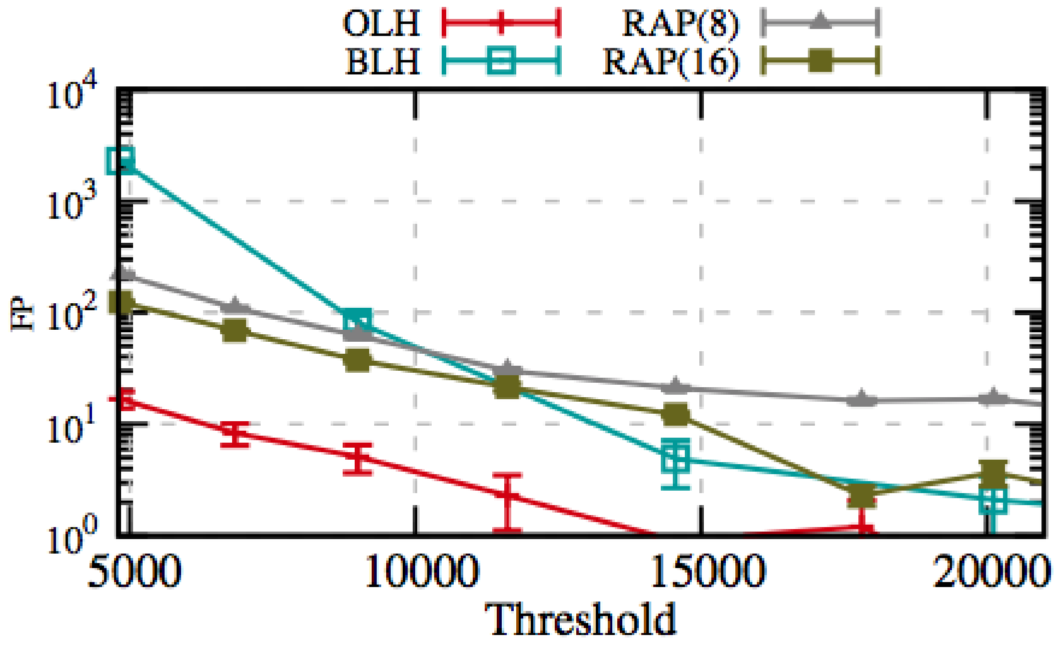
Backup: Accuracy on Frequent Values



Backup: On Information Quality



(a) Number of True Positives



(b) Number of False Positives

Backup: On answering multiple questions

- Previously works (including centralized DP) suggest splitting privacy budget
- For example, when a user answers two questions, privacy budgets are $\varepsilon/2$ and $\varepsilon/2$ (assuming the two questions are of equal importance)
- In the centralized setting, there are sequential composition and parallel composition
  - By partitioning users, one uses to parallel composition
  - By split privacy budget, one uses sequential composition
  - The two can basically produce equivalent results
- What about the local setting?

Backup: On answering multiple questions

- Measure the frequency accuracy (normalize since two approach have different number of users)
  - Assuming OLH is used: $Var[c(v)/n] = \frac{q \cdot (1-q)}{n \cdot (p-q)^2} = \frac{4e^{\varepsilon}}{n \cdot (e^{\varepsilon}-1)^2}$
- Two settings:
  - Split privacy budget: $Var[c(v)/n] = \frac{4e^{\varepsilon/2}}{n \cdot \left(e^{\varepsilon/2}-1\right)^2}$
  - Partition users: $Var\left[c(v)/\frac{1}{2}n\right] = \frac{8e^{\varepsilon}}{n \cdot (e^{\varepsilon}-1)^2}$ ✓
- Algebra shows that it is better to partition users

Thanks to my coauthors

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