Inf2Guard: An Information-Theoretic Framework for Learning Privacy-Preserving Representations against Inference Attacks

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Machine learning



Machine learning As a service



Hazard: Privacy leakage



Membership Inference Attacks (MIA)





Data Reconstruction Attack (DRA)



Defense against inference attacks using Inf2Guard

• Can we design a unified privacy protection framework against these inference attacks, MIA, PIA and DRA, that also **maintain utility?**

 Under the framework, can we further theoretically understand the utility-privacy tradeoff and the privacy leakage against the inference attacks?

Threat Model

Defender objective:

• Learning data representations that are resistant to inference attacks



Inf2Guard

- Inf2Guard is inspired by **information theory** and designs customized mutual information (MI) objectives for each inference attack.
- Goal 1: Privacy protection
- Goal 2: Utility preservation.

Introduction to Mutual Information (MI)

- It measures the **amount of information** that one random variable *X* provides about another random variable *Y*.
- **MI quantifies** the reduction in uncertainty about one variable due to the knowledge of the other.
- Mathematically, MI is expressed as:

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$
$$I(X;Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) log^{\left(\frac{p(x,y)}{p(x)p(y)}\right)}$$

• Applications: Used in feature selection, clustering, privacy-preserving mechanisms and inference attack defenses.

How to defense against this MIA?

- Decrease the utility
 - DP-SGD
 - DP-Encoder

- **Does not** have privacy guarantees
 - AdvReg
 - NeuGuard
 - ...

Inf2Guard against MIAs

• Goal 1: Membership protection

Very Challenging to solve these two MI objectives. Calculating an MI between arbitrary variables is infeasible.



• Goal 2: Utility preservation

 $\max_{f} I(y;\mathbf{r}|u=1),$

 $\min_{\mathbf{r}} I(\mathbf{r}; u),$



How to address intractable MI calculation?

- Inspired by the **MI neural estimation**, which transfers the intractable MI calculations to the **tractable variational MI bounds**.
- Capable of parameterizing each bound with a (deep) neural network.
- Train neural networks to approximate the **true MI** and **learn representations** against the inference attacks.

Estimating MI via tractable bounds

 $[q_{\psi}(u|r)]$ is an auxiliary posterior distribution of p(u|r)

• Minimizing the upper bound MI in Equation

$$\begin{split} I(r;u) \leq & I_{vCLUB}(r;u) = E_{p(r;u)} \big[log q_{\psi}(u|r) \big] - E_{p(r)p(u)} \big[log q_{\psi}(u|r) \big] \\ & min_{\Psi} E_{p(r;u)} \big[log (u|r) \big] - E_{p(r)p(u)} \big[log q_{\psi}(u|r) \big] \\ & \Leftrightarrow max_{\Psi} E_{p(r;u)} \big[log q_{\psi}(u|r) \big] \end{split}$$

• Goal 1: privacy protection as a min-max objective function:

 $E_{p(r;u)}[logq_{\psi}(u|r)]$ is irrelevant to Ψ .

 $\min_{f} \min_{\Psi} I_{\nu CLUB}(r; u) \Leftrightarrow \min_{f} \max_{\Psi} E_{p(r; u)} [log q_{\psi}(u|r)]$

Adversarial game between an adversary q_{ψ}) and encoder f

Estimating MI via tractable bounds

• Maximizing the lower bound MI in Equation

 q_{Ω} is an arbitrary auxiliary posterior distribution. Predict the training data label y from the representation r

$$\begin{split} &I(y;r|u=1) = H(y|u=1) - H(y|r,u=1) \\ &= H(y|u=1) + E_{p(y,r,u)} \left[log^q \Omega^{(y|r,u=1)} \right] \\ &\geq H(y|u=1) + E_{p(y,r,u)} \left[log^q \Omega^{(y|r,u=1)} \right] \end{split}$$

• Goal 2: utility preservation can be rewritten as max-max objective function:

$$max_{f}I(y;r|u=1) \Leftrightarrow max_{f}max_{\Omega}E_{p(y,r,u)}\left[log^{q}\Omega^{(y|r,u=1)}\right]$$

Cooperative game between the **encoder f** and $q\Omega$

Objective function of Inf2Guard against MIAs.

 $\lambda \in [0,1]$ tradeoffs privacy and utility

• Our objective function of learning privacy-preserving representations against MIAs:

 $max_{f}(\lambda \min_{\Psi} - E_{p(x,u)}\left[log^{q\Psi(u|f(x)}\right] + (1-\lambda)max_{\Omega} E_{p(x,y,u)}\left[log^{q\Omega(y|f(x),y=1)}\right]$

Implementation in practice:

- Three parameterized neural networks
- Encoder f
- Membership protection network g_{ψ}
- Utility preservation network $oldsymbol{h}_\Omega$

Inf2Guard- Utility Training



Inf2Guard-Attack Training





Theoretical Results

Theorem 1 (Privacy Leakage Bound)

• **Key Result**: The probability that an MIA correctly infers membership u is bounded by:

$$\Pr(A_{MIA}(r) = u) \le 1 - \frac{H(u|r)}{2log_2^{(\frac{6}{H(u|r)})}}$$

- Implication: A larger H(u|r) (conditional entropy) means a lower MIA accuracy, indicating better privacy protection.
- Goal:
 - **Objective**: Maximize H(u|r) by minimizing I(u; r) (Mutual Information), thereby reducing MIA effectiveness.

Experimental results- MIA

• Utility-privacy results

λ	Utility	MIA Acc	
0	78.9%	70.1%	
0.25	78.2%	55.9%	
0.5	78%	53.5%	
0.75	77.2%	51.1%	
1	20%	50%	

• TPR vs FPR of Inf2Guard against LiRA



CIFAR10

Experimental results- MIA

• 3D t-SNE embeddings results on the learnt representation of on CIFAR10.



Experimental results - PIA

• Comparing with the DP-based defense

Defense	Census	
	Utility	PIA Acc
DP-encoder	52%	34%
Inf ² Gaurd	<mark>76%</mark>	34%

• 3D t-SNE embeddings results



Experimental results - DRA



Conclusion

- **Inf2Guard:** A unified information-theoretic framework for learning privacy-preserving representations.
 - Membership Inference, Property Inference, Data Reconstruction
- Guaranteed privacy leakage
- Guaranteed utility-privacy tradeoff
- State-of-the-art of the utility-privacy tradeoff

Contribution

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